PATENT SPECIFICATION

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PROVISIONAL SPECIFICATION

Improvements relating to Mechanical Displacement—Magnifying Systems

We, NATIONAL RESEARCH DEVELOPMENT CORPORATION, a British Corporation, of 1, Tilney Street, London, W.1, do hereby declare the nature of this invention to be as follows:—

This invention relates to a mechanical system which makes use of an element having negative stiffness to obtain the effect of a high-ratio lever.

The basic principle of the system is illustrated in the accompanying diagram. Three spring members, for example open cooled springs at S_1 and S_2 and a "negative" spring at S_3, are connected mechanically in series and the extreme ends of the system are fixed to the common frame E.

Is the numerical value of the stiffness of the negative spring S_3 is correctly related to the total stiffness of S_1 and S_2 (treated as one continuous spring) the system is stable and also has the property that longitudinal displacements of the junction of S_1, and S_2, i.e. the point A, are duplicated by much larger longitudinal displacements of the junction of S_2 and S_3, i.e. the point B.

The ratio of magnification so obtained is governed only by the relative magnitudes of the stiffnesses of S_1, S_2 and S_3, and not by the absolute magnitude of the three quantities.

It is possible therefore to vary the magnification of the assembly by altering the relative magnitudes of the stiffnesses of S_1 and S_2: the larger S_2 becomes relative to S_1 the lower the magnification. The larger S_1 becomes relative to S_2 the higher the magnification, but the greater the apparent stiffness measured at A.

In one simple form the system consists of a continuous coiled spring (S_1+S_2) coupled to a "sine-spring" such as is described in the provisional or complete specification of Patent Application No. 32770/55 (Serial No. 617,076), the sine spring forming the negative stiffness S_3. The input deflection is applied to a tap on the coiled spring by inserting inside the spring a threaded plug meshing with the spring coils. This plug carries an extension piece which passes down the centre of S_1 to the exterior: to this the input deflection is applied. The output deflection appears as motion of point B, which may carry a pointer, a very light frictionless lever, mirror or other device for giving an indication of the motion. S_1 with S_2 may be replaced by a "blade" spring or a combination of springs, the point A representing a point on one of these springs.

It is important to notice that no interference can be tolerated at point B: for instance a "clock" gauge cannot be applied to B: this would mean that B was operating against an unknown and variable stiffness. The movement of B would then be unpredictable and variable to a similar extent.

The system is mechanically neat because of the straight-line layout which involves no friction as to pivots or levers are employed. Internal friction in the "sine-spring" can be corrected for or compensated.

Magnification ratios of 100/1 or more should be possible.

If D_A is the displacement of point A due to an external force applied, D_B the resulting displacement of B, and N is the magnification ratio, i.e. DB/DA. Then by simple mechanical principles it can be shown that:—

$$\frac{S_3}{S_2}=\frac{1}{N}-1$$ (S_3 is a negative quantity) -............ Eqn. 1.

or $$S_2=\frac{N}{1-N}S_3$$ .... Eqn. 2.

This shows that S_3 will be numerically of the order of S_2 if N is large; but S_2 is always $>S_3$.

In addition the stiffness met by the force

[*Price* 2/8]

applied at A (designated S_A) will be given by

$$S_A = S_1 + N\,S_3 \quad \text{.......................... Eqn. 3.}$$

After A has been deflected due to a force being applied A can be considered as locked to the frame. For in practice the point A will be caused to move by an external rigid body being brought to bear against it, and so long so this is the case the point A can be considered as rigidly connected to the frame. The stiffness measured at B is then independent of S_1 and depends only upon S_2 and S_3. The stiffness at B (designated $S^1{}_B$) under these conditions becomes:—

$$S^1{}_B = \frac{S_3}{1-N} = \frac{S_2}{N} \quad \text{.............. Eqn. 4.}$$

$S^1{}_B$ may be called the "output stiffness".

In practice the negative stiffness S_3 can be varied continuously over a wide range. However it is not easy to provide, in a small space negative stiffnesses much greater (numerically) than 1×10^6 dynes/cm.

Since the stiffness S_2 and S_3 are linked by the value of N (as shown above), the choice of S_2 is fixed at just above 1×10^6 dynes/cm. In practice it will be sufficient to choose a spring (S_2) which has a stiffness lying between 0.75 and 1.25×10^6 dynes/cm. Adjustment can be made to stiffen a spring by reducing its length till the desired value is obtained. The stiffness of S_3 can then be adjusted to conform to Eqn. (1) or (2).

It will be seen from Eqn. (3) that the stiffness S_A is made positive or negative by the choice of S_1. It is of course essential that the stiffness S_A be made positive otherwise the system is unstable when free.

As S_1 does not appear elsewhere in the equations, any desired value may be chosen and thus adjust S_A to any required positive value.

The magnitude of the stiffness $S^1{}_B$ is under control, even if N is already fixed. Eqn. (4) shows this, and makes it clear that the use of excessively small numerical values for S_3 (and therefore for S_2) will lead to a system possessing a very small output stiffness $S^1{}_B$. Such a system would be uncertain in action due to its sensitiveness to any unestimated restraint exerted on B by the indicating means fitted at that point, or by internal friction in S_3.

As an illustration there may be considered the problem of designing a "deflection-magnifier" in which $N = 100$

with $S_3 = -1 \times 10^6$ dynes/cm.
and $S_A = 1 \times 10^5$ dynes/cm.

It is seen at once that choice of S_3 and N fixes

$$S^1{}_B \text{ at } \frac{S_3}{1-N}$$

$$\doteq 1 \times 10^4 \text{ dynes/cm.}$$

$$\doteq 1 \text{ oz/inch.}$$

The choice of S_2 is fixed at

$$\frac{N}{1-N} S_3 = \frac{100}{99} \times 10^6 \text{ dynes/cm.}$$

Since S_A is defined as 1×10^5 dynes/cm.,

$$S_1 = (1 \times 10^5) + 10^8$$
$$= 100.1 \times 10^6 \text{ dynes/cm.}$$

It will be noted S_1 will in general always exceed S_A by a very large amount. Hence, from Eqns. (2) (3) and (4) $\frac{S_1}{S_2} = \frac{-N.S_3}{N.S_3}(1-N) \doteq N$. If therefore the point A is merely a "tap" on a long coiled spring, and N is large, the relative lengths of the portions EA and AB will be 1/N or (in the case worked above) will be 1/100th.

As already indicated a "sine-spring" is preferably employed to provide the negative stiffness required at S_3: it has the property of continuous adjustment so that the close adjustment of negative stiffness required by Eqn. 2 can readily be made. It also has the important property that when it is set in its symmetrical position it has a numerically maximum negative stiffness. Any deflection from this position makes S_3 numerically smaller; as a result S_A can never become negative (see Eqn. (3)) and the system unstable. However, the constancy of S_3 with deflection of B must be high if N is to remain constant.

In a practical design the magnification ratio N will therefore be a maximum when the "sine-spring" is symmetrically disposed and falls off as the spring is deflected. If S_3 decreases numerically as B moves we see from Eqn. (2) (converted to $N = S_2/(S_2 + S_3)$) that N will become smaller.

However, it should be possible to compensate for this effect by making S_2 fall slightly as B moves: for instance S_2 may contain a few turns which are in initial compression, or S_2 may be paralleled by a light spring all of whose turns are in compression. If the compensation is overdone, however, the system will become unstable.

The invention is regarded as of general application where means for magnifying mechanically a small displacement is required. One specific application is in indicating micrometers, e.g. of the clock-gauge type, the input small deflection being applied at "A" and the large output deflection appearing at B.

Dated 4th day of November, 1948.

S. W. SLAUGHTER,
Agent for the Applicants.

COMPLETE SPECIFICATION

Improvements relating to Mechanical Displacement—Magnifying Systems

We, NATIONAL RESEARCH DEVELOPMENT CORPORATION, a British Corporation, of 1, Tilney Street, London, W.1, do hereby declare the nature of this invention and in what manner the same is to be performed, to be particularly described and ascertained in and by the following statement:—

This invention relates to systems for the magnification of small displacements and involves the use of a spring element possessing negative stiffness. One example of such an element occurs in British Patent Specification 617,076 wherein is described a spring of sinusoidal form (hereinafter referred to as a "sine-spring") mounted between adjustable anchorages so that its stiffness can be made positive, zero or negative as desired.

To facilitate the understanding of this invention reference is made initially to the drawing left with the provisional specification. In this drawing three spring members, for example open coiled springs at S_1 and S_2 and a "sine-spring" at S_3, are connected mechanically in series and the extreme ends of the system are fixed to a common frame E.

If the numerical value of the stiffness of the negative spring S_3 is correctly related to the total stiffness of S_1 and S_2 (treated as one continuous spring) the system is stable and also has the property that longitudinal displacements of the junction of S_1 and S_2, i.e., the point A, are duplicated by much larger longitudinal displacements of the junction of S_2 and S_3, i.e., the point B.

The ratio of magnification so obtained is governed only by the relative magnitudes of the stiffnesses of S_1, S_2 and S_3, and not by the absolute magnitude of the three quantities.

It is possible therefore to vary the magnification of the assembly by altering the relative magnitudes of the stiffnesses of S_1 and S_2; the larger S_2 becomes relative to S_1, the lower the magnification. The larger S_1 becomes relative to S_2 the higher the magnification, but the greater the apparent stiffness measured at A.

According to the invention a system for the magnification of small displacements comprises a spring arrangement of positive stiffness in combination with a spring arrangement of negative stiffness, the springs being in series and their relative stiffness being such that a displacement or input deflection intermediate the ends of the spring arrangement of positive stiffness results in a magnified displacement or output deflection at the junction of the spring arrangement of positive stiffness with the spring arrangement of negative stiffness.

The aforesaid spring arrangement of positive stiffness may be either a single cantilever spring member or two springs of positive stiffness in series.

Where a single cantilever spring member is used, the said member may be coupled at a point at or near its free end (the point B previously referred to) to a "sine-spring" arranged to offer negative resistance in the direction of displacement of the said cantilever spring member, the point at which the input deflection is applied (the point A previously referred to) being a point along its length preferably close to its clamped end.

The two springs of positive stiffness in series may be constituted by a continuous spring coil having a tap defining the point at which the input deflection is applied (point A) the said spring coil being coupled to a "sine-spring" arranged for negative stiffness. The tap may be formed by inserting inside the coil a threaded plug meshing with the coil and carrying an extension piece protruding through the spring at a point along its length.

In both of the foregoing particular forms of the invention the output deflection appears as a motion of the coupling point of the "sine-spring" to which may be attached a pointer, very light frictionless lever, mirror or other device for giving an indication of the motion. It is important to note that no interference can be tolerated at the point B; for instance a dial gauge cannot be applied to B because the point would then be operating against an unknown and variable stiffness and its movement would be unpredictable.

Reference will now be made to the accompanying drawing in which Figures 1 and 2 are respectively a side view and a part section on the line II II of Figure

1 of a preferred construction of instrument embodying the system according to the invention.

The essential components of the instrument are a cantilever blade spring 1 forming a pointer and coupled to a " sine-spring " 2 by a thin flexible blade 3 crimped or riveted at each end. A thin flexible blade is used as a coupling member to avoid any friction which would be incurred by the use of a link and pin joints. The components are housed in a casing 7.

The blade 1, the free end of which moves over a scale 1*a*, is clamped in a slot in a relatively thick blade 4 after being bent as shown in Figure 1. The blade 4 is secured to a raised part of the casing 7 by screws 4*a*. Adjustment for zero error of the blade 1 may be made by rotation of a screw 5 having a grooved end engaging a slot 6 in the blade 4. Rotation of the screw 5 bends the blade 4 and causes a corresponding deflection of the blade 1. Provision of this zero error adjustment is preferred since the effective stiffness of the blade 1 should be approximately zero when coupled to the " sine-spring 2 " and it is thus sensitive to very small forces, e.g., as applied to it when the instrument is tipped. The displacements to be measure are applied to a pin 8 which bears on a hardened stud 9 secured to the blade 1. An adjustable stop 10 is provided to prevent excessive displacement of the pin 8 which might result in damage to the blade 1. Alternatively the pin 8 might be at right angles to the position shown and bear on an unbent blade. In this alternative arrangement a similar zero adjustment to that already described would be provided. The instrument may be supported in the usual way by an arm 11 and clamp 12.

If the blade 1 is uniform, its length from its fixed end to the coupling point of the " sine spring " is L, and the deflecting force is applied at a distance d from its fixed end, then the magnification ratio N is of the order of $(L/d)^2$. (Assuming that the negative stiffness of the " sine-spring " 2 is very nearly equal and opposite to the stiffness of the blade 1 measured at the coupling point). It is thus desirable to make L large, i.e., by coupling the " sine-spring " as near to the pointer as possible, and d small, i.e., by applying the deflecting force close to the fixed end of the blade 1. However, if the blade 1 is not uniform, or in the general case, S_1 and S_2 are not similar types of springs, it is possible to obtain a value for N greater than $(L/d)^2$. For instance, if the blade 1 is made 8 times as wide (or twice as thick) from its fixed end to the stud 10 and then reduced in section for the remainder of its length up to the coupling point of the " sine-spring " then if d_2 is the length of the part of blade 1 of larger section, the magnification ratio becomes $8\ (L/d_2)^2$.

The setting of the " sine-spring " 3 preferred is with its end clamped in line, as shown in Figure 1. In this setting the negative stiffness of the spring remains substantially constant for deflections not greater than half the amplitude of the curve formed by one " bow " of the spring. The calibration marks on the scale of the instrument will therefore be nearly evenly spaced over this range of movement of the pointer. If the deflection is large enough to cause the " sine-spring " 2 to be deflected by more than half of the " bow " of the spring (or if the ends of the spring 2 are not exactly in line), the magnification factor will vary over the scale. For some purposes this may be advantageous. For instance the scale shown in Figure 1 while providing for a large range of input deflection by compression at each end of the scale, has the centre marks well separated for precision reading.

The magnification factor of the instrument can be adjusted by altering the angles of anchorages 2a, 2b of the " sine-spring " 2. This allows wide tolerances in the material and dimensions of the blade 1 and simplifies production.

Reverting to a consideration of the drawing left with provisional specification, if D_A is the displacement of point A due to an external force applied, D_B the resulting displacement of B, and N is the magnification ratio, i.e., D_B/D_A. Then by simple mechanical principles it can be shown that:—

$$\frac{S_3}{S_2}=\frac{1}{N}-1 \quad (S_3 \text{ is a negative quantity}) \quad \text{............ Eqn. 1.}$$

$$\text{or } S_2=\frac{N}{1-N}\,S_3 \quad \text{............... Eqn. 2.}$$

This shows that S_3 will be numerically of the order of S_2 if N is large; but S_2 is always $> S_3$.

In addition the stiffness met by the force applied at A (designated S_A) will be given by

$$S_A=S_1+N\,S_3 \quad \text{..................... Eqn. 3.}$$

After A has been deflected due to a force being applied A can be considered as locked to the frame. For in practice the point A will be caused to move by an external rigid body being brought to bear against it, and so long so this is the case the point A can be considered as rigidly connected to the frame. The stiffness measured at B is then independent of S_1 and depends only upon S_2 and S_3. The stiffness at B (designated S^1B) under these conditions becomes:—

$$S^1{}_B = \frac{S_3}{1-N} = \frac{S_2}{N} \quad \ldots\ldots\ldots\ldots\ldots \text{ Eqn. 4.}$$

$S^1{}_B$ may be called the "output stiffness".

In practice the negative stiffness S_3 can be varied continuously over a wide range. However it is not easy to provide, in a small space negative stiffnesses much greater (numerically) than 1×10^6 dynes/cm.

Since the stiffness S_2 and S_3 are linked by the value of N (as shown above), the choice of S_2 is fixed at just above 1×10^6 dynes/cm. In practice it will be sufficient to choose a spring (S_2) which has a stiffness lying between 0.75 and 1.25×10^6 dynes/cm. Adjustment can be made to stiffen a spring by reducing its length till the desired value is obtained. The stiffness of S_3 can then be adjusted to conform to Eqn. (1) or (2).

It will be seen from Eqn. (3) that the stiffness S_A is made positive or negative by the choice of S_1. It is of course essential that the stiffness S_A be made positive otherwise the system is unstable when free.

As S_1 does not appear elsewhere in the equations, any desired value may be chosen and thus adjust S_A to any required positive value.

The magnitude of the stiffness $S^1{}_B$ is under control, even if N is already fixed. Eqn. (4) shows this, and makes it clear that the use of excessively small numerical values for S_3 (and therefore for S_2) will lead to a system possessing a very small output stiffness $S^1{}_B$. Such a system would be uncertain in action due to its sensitiveness to any unestimated restraint exerted on B by the indicating means fitted at that point, or by internal friction in S_3.

As an illustration there may be considered the problem of designing a "deflection-magnifier" in which $N = 100$

with $S_3 = -1 \times 10^6$ dynes/cm.
and $S_A = 1 \times 10^5$ dynes/cm.

It is seen at once that choice of S_3 and N fixes

$$S^1{}_B \text{ at } \frac{S_3}{1-N}$$
$$\doteqdot 1 \times 10^4 \text{ dynes/cm.}$$
$$\doteqdot 1 \text{ oz/inch.}$$

The choice of S_2 is fixed at

$$\frac{N}{1-N} S_3 = \frac{100}{99} \times 10^6 \text{ dynes/cm.}$$

Since S_A is defined as 1×10^5 dynes/cm.,

$$S_1 = (1 \times 10^5) + 10^8$$
$$= 100.1 \times 10^6 \text{ dynes/cm.}$$

It will be noted S_1 will in general always exceed S_A by a very large amount. Hence, from Eqns. (2) (3) and (4) $\frac{S_1}{S_2} \doteqdot \frac{-N.S_3}{N.S_3}(1-N) \doteqdot N$. If therefore the point A is merely a "tap" on a long coiled spring, and N is large, the relative lengths of the portions EA and AB will be 1/N or (in the case worked above) will be 1/100th.

As already indicated a "sine-spring" is preferably employed to provide the negative stiffness required at S_3: it has the property of continuous adjustment so that the close adjustment of negative stiffness required by Eqn. 2 can readily be made. It also has the important property that when it is set in its symmetrical position it has a numerically maximum negative stiffness. Any deflection from this position makes S_3 numerically smaller; as a result S_A can never become negative (see Eqn. (3)) and the system unstable. However, the constancy of S_3 with deflection of B must be high if N is to remain constant.

In a practical design the magnification ratio N will therefore be a maximum when the "sine-spring" is symmetrically disposed and falls off as the spring is deflected. If S_3 decreases numerically as B moves we see from Eqn. (2) (converted to $N = S_2/(S_2 + S_3)$) that N will become smaller.

However, it should be possible to compensate for this effect by making S_2 fall slightly as B moves: for instance S_2 may contain a few turns which are in initial compression, or S_2 may be paralleled by a light spring all of whose turns are in compression. If the compensation is overdone, however, the system will become unstable.

The invention is regarded as of general application where means for magnifying mechanically a small displacement is required. One specific application is in indicating micrometers, e.g. of the dial-gauge type, the input small deflection being applied at "A" and the large output deflection appearing at B.

Having now particularly described and ascertained the nature of the said invention and in what manner the same is to be performed, we declare that what we claim is:—

1. A system for the magnification of small displacements comprising a spring arrangement of positive stiffness in combination with a spring of negative stiffness, the springs being in series and their relative stiffness being such that a displacement or input deflection intermedi-

ate the ends of the spring arrangement of positive stiffness results in a magnified displacement or output deflection at the junction of the spring arrangement if positive stiffness with the spring of negative stiffness.

2. A system according to Claim 1 wherein the spring arrangement of positive stiffness comprises a single cantilever spring member coupled at or near its free end to a " sine-spring " arranged to offer negative stiffness in the direction of displacement of the said cantilever spring member, the input deflection being applied at a point intermediate the ends of the said cantilever spring member.

3. A system according to Claim 2 in which the free end of the cantilever spring member moves over a scale to indicate the output deflection.

4. A system according to Claim 2 or Claim 3 in which the cantilever spring member is clamped in a relatively stiff blade member which can be displaced by screw or like means so that adjustment is effected for zero error displacement.

5. A system according to any one of Claims 2 to 4 having the part of its length between its fixed end and the point of application of the input deflection formed of increased cross-section.

6. A system according to Claim 1 comprising a continuous coiled spring coupled to a " sine-spring ", the input deflection being applied at a point intermediate the ends of the said continuous coiled spring.

7. A system according to Claims 2, 3, 4 or 5 in which the " sine-spring " is clamped with its ends in line.

8. A system as claimed in Claim 1 constructed and arranged for operation substantially as described with reference to the drawings herein referred to.

Dated this 1st day of November, 1949.
S. W. SLAUGHTER,
Agent for Applicants.

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Published at The Patent Office, 25, Southampton Buildings, London, W.C.2, from which copies may be obtained.

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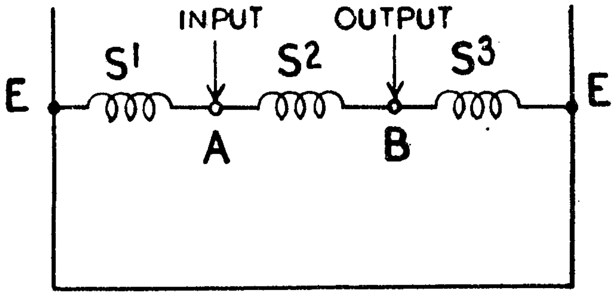

674,856 COMPLETE SPECIFICATION
1 SHEET

This drawing is a reproduction of the Original on a reduced scale.

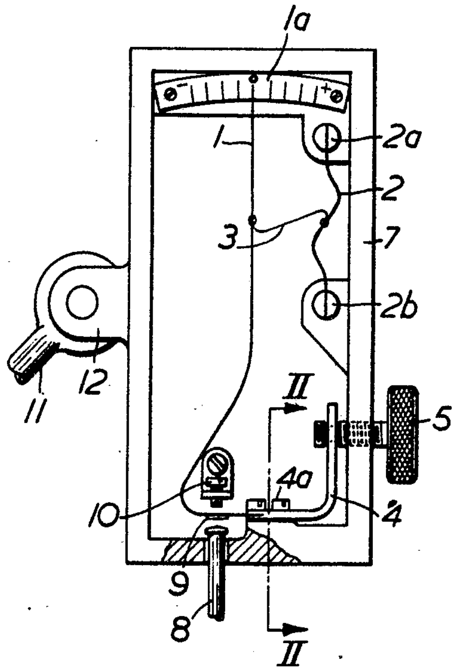

Fig. 1

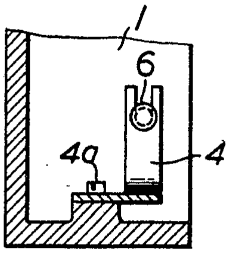

Fig. 2