# PATENT SPECIFICATION

630,243

Application Date: Dec. 29, 1947. No. 34551/47.

Complete Specification Left: March 11, 1949.

Complete Specification Accepted: Oct. 7, 1949.

Index at acceptance:—Classes 97(iii), H2bx; and 106(ii), H2d.

PROVISIONAL SPECIFICATION

## Linkage for the Generation of Involute-like Curves

I, EDMUND RAMSAY WIGAN, a British Subject, of 35, Montague Road, Southbourne, Bournemouth, Hampshire, do hereby declare the nature of this invention to be as follows:—

This invention relates to a very simple linkage which is capable of tracing curves which approximate so closely to a true involute, over the initial part of the arc, that they can be substituted, without serious error, for true involute curves in the draughting of drawings of involute-type spur-gear teeth.

By obvious substitutions the same linkage may be employed as an involute-checking or gauging mechanism.

The closeness with which the generated curves approximate a true involute can be varied to some extent so that, if the tooth is very short, in proportion to the wheel radius, a very close approximation can be maintained for its full length or, again, if the tooth is relatively long, the approximation can be made less perfect near the origin but exact at a point at or near the tip of the tooth.

The basic geometrical principle is best illustrated by an example:—Let O be the centre of the involute base-circle of radius R. Let T be the point on the base-circle at which the tooth profile is required to originate. Draw a circle of centre O and radius 0.810.R. Produce TO to cut this circle at P. Now through T draw a series of lines all of length PT, originating from points P1, P2, etc., on the circumference of the inner circle and inclined at a variety of angles to the line POT and on one side of it. Then the ends of these lines (designated $t1$, $t2$, $t3$, etc.), lying outside the base-circle, will lie on a curve which, near its origin, is a very close approximation to an involute of origin T and base-circle radius R.

The calculation of the degree of approximation is extremely tedious, but it can be shown by straight-forward tabulation that (if we write $Tt/R.=x$, and if we write KR as the radius of the inner circle), then, with K=0.810 the approximation is perfect at the point on the curve at which $x=0.150$, and at any intermediate point the deviations of the points $t1$, $t2$, $t3$, etc., from the true involute originating at T do not exceed thirty parts in a million of the radius R.

For longer teeth the choice of K=0.815 gives perfect fit when $x=0.2$ approx., with a maximum error of less than 100 parts in a million of the radius R about halfway along the tooth.

The calculation of the errors for the cases in which the geometrical conditions are not exactly as laid down above is extremely tedious and exact figures cannot be given. But graphical tests show that, for teeth in which $x$ considerably exceeds 0.2, it is possible to get close approximation up to the tip of the tooth by applying slight modifications to the geometric construction.

For instance, let a point S be chosen some 10 or 20 degs. to the left of the tooth-origin T and on the base-circle. Imagine a piece of metal cut to the shape of the triangle PST; call the apices of this metal piece P1, S1, T1. Then, if as P1 moves round the inner circle to the left of P1, the side P1.S1 of this triangle is made always to intersect point S on the base-circle, then the apex T1 will trace out the approximate involute curve. Values of K lying between 0.8 and 0.9 appear to give the closest approximation to a true involute (when $x$ is greater than 0.2) and when S lies some 10 or 20 degs. away from T.

There would appear to be some theoretical advantage in placing the point S not on the base-circle but slightly inside it, say at 0.9R. In practice however this would add another adjustment to a draughting machine based upon this geometry, the complication of which

[*Price* 2/-]

would possibly not justify its inclusion.

The application of these geometric principles to a draughting machine leads to a generic description as follows:— Means for tracing a curve approximately representing the profile of an involute spur-gear tooth comprising a first rigid member pivoted on a support located over the centre of the involute base-circle, coupled by a pivot to a second rigid member (both members being adjustable in length to suit the base-circle radius), a device, adapted to hold a pen or pencil, secured to the second member and means secured to the drawing surface provided with a pivoted guide-piece engaging with the second member, the proportions and dispositions of the parts being such that when the first member is rotated about its fixed support the pencil-carriage traces an arc closely approximating to the initial part of an involute curve.

One form of such a draughting machine would consist of a plate provided with cross-wires or other means of locating its centre over the centre, O, of the base-circle, and carrying the first member mounted upon a suitable pivot or bearing. The said member would be provided with an adjustable clamp to which the second member would be secured by a pin, or the like, to form a freely turning hinge, the distance of the hinge from the fixed bearing being indicated on some kind of scale of inches, centimeters, etc., associated with the first member.

The second member would be held, near one end, in the said adjustable clamp, and the other end would pass through a pivoted guide-piece turning in a frame secured to the drawing surface (by drawing-pins or the fingers of the user, for instance), at the point on the drawing required to conform to the geometrical principles set out above: for instance, in the simple case, when S and T coincide, so that the pivot of the guide piece was centred over the origin, T, of the required tooth-profile.

The pen- or pencil-carriage would, preferably, be secured to the second member in such a way that, while being rigidly restrained against motion in the plane of the paper, it would be free to move vertically under the influence of a spring-like member which would press the pencil onto the paper.

The way in which the mechanism is set up for use is illustrated below by a description of the procedure suitable for the case in which the points T and S coincide. Modifications applicable to other geometric conditions are obvious from the details set out above.

The cross-wires on the main bearing plate is located over the centre of the base-circle. The point of origin, T, of the required tooth-profile is marked on the drawing. The first and second members are aligned so as to lie along the diameter of the base-circle passing through T. The frame carrying the second member guide-piece is adjusted so that the centre of the pivot lies exactly over T (to ensure this the pivots might be removed and the pivot-holes used as a sighting-line). The adjustable clamp associated with the first and second member is released and the pencil-point is placed on the mark T. The clamp is adjusted so that the pivot of the clamp lies at the correct radius (K times the base-circle radius), along the first member, and then locked. Rotation of the first member will then cause the pencil to trace out the required curve: either of the two involutes arising at T can be approximated by the choice of either L.H. or R.H. direction of rotation of the first member about the primary fixed pivot.

When the linkage is set up correctly the curves traced deviate very slightly from the true involute to an extent invisible to the naked eye when the wheel radius is small and the teeth short, yet, in drawing long teeth on large spur-wheels, visible errors may occur unless additional adjustments are made to the linkage. These errors tend either to increase steadily as the first member is rotated, or to increase to a maximum, fall to zero and then pass to steadily increasing magnitudes of reversed sign. In the case of S and T coinciding, for example, the first type of error occurs if K is less than 0.800 approx., and the latter if K is of the order 0.8 to 1.0. It is obvious that the substitution of an eccentrically mounted pivot in place of the guide-piece controlling the motion of the second member will, with the proper degree and alignment of the eccentricity, be capable of correcting errors of both types; alternatively the pin coupling the first and second members may, by additional members added to the linkage, be caused to change its position on the first member; the radius from the point O might be varied as rotation increases: or the pin might be mounted eccentrically on a part of the member which rotated relative to the main member when the latter was rotated about its primary fixed pivot. The most promising of these fine adjustments to the linkage would appear to be one associated with the second member guide-piece, since this piece rotates as the second member slides through it and therefore the means, lacking in the other methods, for causing

relative rotation of a pivot is to hand. The principle of such a method of adjustment of the curve traced can be seen if we consider one particular arrangement of the linkage in which the guide-piece consists of a pin pierced with a hole to accept the second member which slides through this hole; suppose that, without the assistance of any eccentric, the curve was, for a particular conformation of the linkage, lying on the inner side of the true involute, then a slight movement of the pivot of the guide-piece along the line OT would bring curve and involute into coincidence (at the one point). Such a motion of this pivot is readily derived by carrying the pivot on a sub-plate eccentrically supported on the frame carrying the guide-piece assembly.

Alternatively the second member may be constrained to slide over a suitably curved surface rather than pass through a pivoted guide-piece as described above.

The first member may have associated with it a scale or scales marked off in units of "K-inches," or "K-cms.," so that the diameter of the base-circle may be used as the mark on this scale to which the adjustable clamp is set; this relieves the draughtsman of any calculation. Alternatively the second member may be marked off in "(1 plus K) inches," or cms., etc.

Dated this 26th day of December, 1947.

E. RAMSAY WIGAN.

## COMPLETE SPECIFICATION

# Linkage for the Generation of Involute-like Curves

I, EDMUND RAMSAY WIGAN, a British Subject, of 35, Montague Road, Southbourne, Bournemouth, Hampshire, do hereby declare the nature of this invention and in what manner the same is to be performed to be particularly described and ascertained in and by the following statement:—

This invention relates to a very simple linkage which is capable of tracing curves which approximate so closely to a true involute, over the initial part of the arc, that they can be substituted, without serious error, for true involute curves in the draughting of drawings of involute-type spur-gear teeth.

By obvious substitutions the same linkage may be employed as an involute-checking or gauging mechanism.

The closeness with which the generated curves approximate a true involute can be varied to some extent so that, if the tooth is very short in proportion to the wheel radius, a very close approximation can be maintained for its full length or, again, if the tooth is relatively long, the approximation can be made less perfect near the origin but exact at a point at or near the tip of the tooth.

According to the present invention a linkage for generating a curve which approximates to an involute curve for a spur-gear tooth comprises a first arm pivoted on a support adapted to be located over the centre of the involute base-circle, a second arm coupled to said first arm by a pivot and adapted to carry a tracing point and a pivoted guide-piece adapted to receive said second arm, the proportions and dispositions of the parts being such that when the said first arm pivots on said support said tracing point generates the required curve.

The invention will be more easily understood from the following description made with reference to the accompanying drawings of which:—

Fig. 1 illustrates the basic geometry of a linkage according to the invention;

Fig. 2 shows one embodiment of the invention;

Fig. 3 illustrates a modification of the invention and Fig. 4 shows a mechanism embodying another modification.

The basic geometrical principle of the invention is illustrated in Fig. 1 in which O is the centre of the involute base-circle of radius R. T is a point on the base-circle at which a tooth profile is required to originate. Consider now an inner circle concentric with the base-circle but of radius KR (where K is about 0.8). The straight line TO produced cuts the inner circle at P. From a series of points $P_1$, $P_2$, . . . a series of lines $P_1t_1$, $P_2t_2$. . . . all of length PT, are drawn; then it can be shown that the ends ($t_1$, $t_2$ . . .) of these lines lying outside the base-circle will lie on a curve which, near the origin, is a very close approximation to an involute of origin T and base-circle radius R.

The calculation of the degree of approximation is extremely tedious, but it can be shown by straight-forward tabulation that, if we write $Tt/R=x$, and if we write KR as the radius of the inner circle, then, with $K=0.810$, the approximation is perfect at the point on the curve at which $x=0.150$, and at any inter-

mediate point the deviations of the points $t_1$, $t_2$, $t_3$, etc., from the true involute originating at T do not exceed thirty parts in a million of the radius R.

For longer teeth the choice of K=0.815 gives perfect fit when $x$=0.2 approx., with a maximum error of less than 100 parts in a million of the radius R about halfway along the tooth. The calculation of the errors for the cases in which the geometrical conditions are not exactly as laid down above is extremely tedious and exact figures cannot be given. But graphical tests show that, for teeth in which $x$ considerably exceeds 0.2, it is possible to get close approximation up to the tip of the tooth by applying slight modifications to the geometric construction.

An embodiment of the invention is illustrated in Figure 2 of the drawings. This embodiment comprises a plate member 1 having an aperture 2 in which there is arranged cross-wires 3. The plate 1 carries a first arm 4 mounted to rotate about a centre O which is the intersection of the cross-wires 3. The arm is slotted and calibrated as shown and carries an adjustable pivot 5 to which a second arm 6 is secured in such a way that the arm 6 is free to pivot with respect to the arm 4 about a pivot $P_1$ which may be fixed at a distance KR (read off the calibrated scale) from the centre O. The arm 6 also passes through a pivoted guide-piece 7 turning in a frame 18 arranged so that the axis of the arm always passes through a point over the point T at a distance R from O (the points O, $P_1$ and T correspond with the similarly indicated points in the theoretical diagram, Fig. 1).

The arm 6 carries at the end remote from the clamp 5 a tracing point 8 which generates the required involute curve as the arm 4 rotates about the point O.

When the mechanism is used as a drawing machine the tracing point may be a pen or pencil and would, preferably, be secured to the second member in such a way that, while being rigidly restrained against motion in the plane of the paper, it would be free to move vertically under the influence of a spring-like member which would press the pencil on to the paper.

In use, the cross-wires 3 are located over O the centre of the base-circle and the point of origin T, of the required tooth-profile is marked on the drawing. The arms 4 and 6 are then aligned so as to be along the diameter of the base-circle passing through T. The frame 18 is adjusted so that the centre of the pivoted guide-piece 7 lies exactly over T (to ensure this the piece 7 might be removed and the pivot-holes used as a sighting line). The clamp 5 is released and adjusted so that the tracing point is on T and the pivoting point of the arm 6 about the arm 4 lies at the correct radius (K times the base-circle radius) and then the clamp 5 is locked. Rotation of the arm 4 will then cause the tracing point to generate the required curve; either of the two involutes arising at T can be generated by the choice of either left-hand or right-hand rotation of the arm 4 about O.

When the linkage is set up correctly the curves traced deviate very slightly from the true involute to an extent invisible to the naked eye when the wheel radius is small and the teeth short, yet, in drawing long teeth on large spur-wheels, visible errors may occur unless additional adjustments are made to the linkage. These errors tend either to increase steadily as the first member is rotated, or to increase to a maximum, fall to zero and then pass to steadily increasing magnitudes of reversed sign.

The apparatus may be modified to reduce these errors still further and one of these modifications will be explained with reference to Fig. 3 of the drawings. In this diagram there is shown a point S chosen some 10 to 20 degrees to the left of the tooth-origin T and on the base-circle. Imagine a piece of metal cut to the shape of the triangle PST; call the apices of this metal piece $P_1$, $S_1$, $T_1$. Then if as $P_1$ moves round the inner circle to the left of P, the side $P_1S_1$ of this triangle is made always to intersect point S on the base-circle, then the apex $T_1$ will trace out the approximate involute curve. Values of K lying between 0.8 and 0.9 appear to give the closest approximation to a true involute (when $x$ is greater than 0.2) and when S lies some 10 or 20 degrees away from T.

There would appear to be some theoretical advantage in placing the point S not on the base-circle but slightly inside it, say at 0.9R. In practice however this would add another adjustment to a draughting machine based upon this geometry, the complication of which would possibly not justify its inclusion.

The mechanical realisation of this modification is obvious, thus the arm 6 of Fig. 2 may be replaced by a triangular member $P_1$, $S_1$, $T_1$ bent at a right angle at $S_1$ and arranged to slide in a pivoted guide-piece similar to 7 in Fig. 2 but located at S which is some 10 to 20 degrees away from the tooth origin T. In this case the part $P_1S_1$ corresponds with the arm 6 and the tracing point is at $T_1$

so that it is off-set from the axis of the arm 6.

The mechanical embodiment of another modification is illustrated in plan view in Fig. 4. In this drawing parts similar to those in Fig. 2 are similarly numbered. It will be seen that the guide-piece 7, instead of being located exactly above the tooth origin T, is mounted eccentrically in a frame 9 which is free to pivot about another point $T_0$ slightly to the right of T. The positions of $T_0$ and T and the dimensions of the parts are such that in the initial position (i.e., with the arms 4 and 6 in line and passing over the point O) the tracing point is at T. As the arm 4 is then moved to the left the arm 6 slides through the guide-piece 7 and at the same time causes it and the frame 9 to pivot about $T_0$.

It is only in the case of teeth of unusual length that the complications of either of these modifications prove worth while, and, even then, only when very close approximation to a true involute is essential. The simple mechanism of Fig. 2 will suffice for most work and subsidiary parts, such as the eccentrically mounted pivot 9, 7 in Fig. 4, would be provided to be used only in exceptional cases.

The arm 4 may conveniently be calibrated in " K-inches " or similar units to relieve the draughtsman of any calculation: it is suggested that two scales, for $K=0.810$ and $K=0.815$, be provided.

Having now particularly described and ascertained the nature of my said invention and in what manner the same is to be performed, I declare that what I claim is:—

1. A linkage for generating a curve which approximates to an involute curve for a spur-gear tooth and comprising a first arm pivoted on a support adapted to be located over the centre of the involute base-circle, a second arm coupled to said first arm by a pivot and adapted to carry a tracing point (e.g., a pen or pencil) and a pivoted guide-piece adapted to receive said second arm, the proportions and dispositions of the parts being such that when the said first arm pivots on said support said tracing point generates the required curve.

2. A linkage according to claim 1, and in which the effective lengths of said arms are adjustable, for example by adjustment of the position of said pivot.

3. A linkage according to claim 1 or 2, and in which the said tracing point is off-set from the axis of said second arm.

4. A linkage according to claim 1 or 2, and in which the said pivoted guide-piece is itself mounted in a frame which is arranged to pivot about a point near the origin of said curve.

5. A linkage substantially as hereinbefore described with reference to Figs. 1 and 2 of the accompanying drawings.

6. A linkage substantially as hereinbefore described with reference to Figs. 1 and 2 but modified as described with reference to Fig. 3 of the accompanying drawings.

7. A linkage substantially as hereinbefore described with reference to Figs. 1 and 2 but modified as described with reference to Fig. 4 of the accompanying drawings.

Dated this 11th day of March, 1949.
E. E. WRIGHT,
Agent for the Applicant.

Leamington Spa: Printed for His Majesty's Stationery Office, by the Courier Press.—1949.
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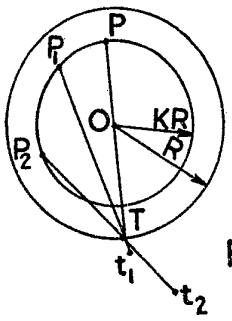


Fig.1.

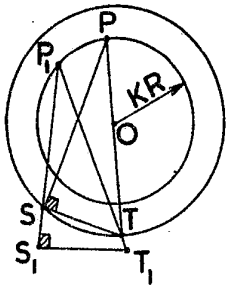


Fig.3.

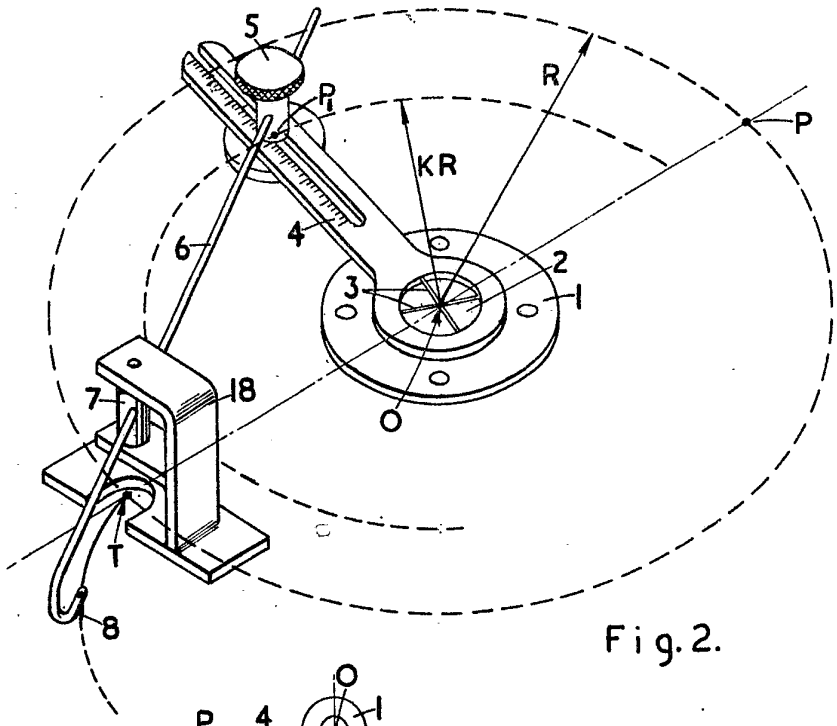


Fig.2.

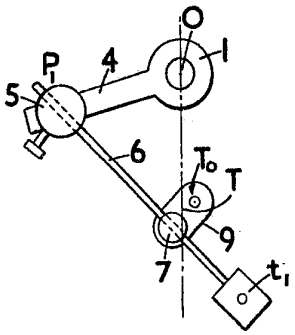


Fig.4.

H.M.S.O. (Ty.P.)